Wiley
Advances in Fuzzy Systems
Volume 2025, Article ID 6739462, 16 pages
https://doi.org/10.1155/adfs/6739462

WILEY

Research Article

Score Function Algorithmic Approach in Optimizing Dual Hesitant Fuzzy Multiobjective Transportation Models

Ritu,[1] **Tarun Kumar**,[1] **Jahnvi**,[1] **Kapil Kumar**,[1,2] **Kailash Dhanuk**,[1,3] **Anirudh Kumar Bhargava**,[4] **Sanjay Kumar Tyagi**,[5] **and M. K. Sharma**[1]

[1]*Department of Mathematics, Chaudhary Charan Singh University, Meerut 250004, Uttar Pradesh, India*
[2]*Department of Mathematics, Atma Ram Sanatan Dharma College (University of Delhi), New Delhi 110021, India*
[3]*Department of Mathematics, Dyal Singh College (University of Delhi), New Delhi 110003, India*
[4]*Department of Mathematics, MMH College Ghaziabad, Chaudhary Charan Singh University, Meerut, Uttar Pradesh, India*
[5]*Department of General Studies (Mathematics), Higher College of Technology, Ras Al Khaimah, UAE*

Correspondence should be addressed to M. K. Sharma; drmukeshsharma@gmail.com

Received 25 March 2025; Revised 6 June 2025; Accepted 30 July 2025

Academic Editor: Ferdinando Di Martino

Multiobjective transportation problem (MOTP) seeks to optimize multiple, often conflicting objectives within transportation and logistics systems. However, in real-world scenarios, parameters such as transportation costs, time, availability, and demand are inherently uncertain. To address these uncertainties, various extensions of fuzzy logic such as intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PFSs), Fermatean fuzzy sets (FFSs), and hesitant fuzzy sets (HFSs) and more have already been applied to MOTPs. Despite these advancements, the dual HFS (DHFS) has not yet been incorporated into MOTP. This research article introduces the novel concept of dual hesitant fuzzy MOTP (DHFMOTP), utilizing DHFS for the first time in this MOTP. A new score function for DHFS and an accompanying algorithm are developed effectively to solve the DHFMOTP. To validate the proposed methodology, two numerical problems are solved. Computed results demonstrate the significant improvements. The proposed score function and algorithm for DHFMOTP consistently yielded more optimal results than the existing methods. Future studies could explore the integration of other advanced fuzzy extensions and hybrid algorithms for further improvement of decision-making efficiency in transportation problems under uncertainty.

Keywords: dual hesitant fuzzy set; fuzzy set; hesitant fuzzy set; multiobjective transportation problem; score function

1. Introduction

Transportation problem (TP) is mostly considered to minimize the transportation cost (TC) from sources to different destinations within the time duration. Numerous real-world decision-making scenarios, such as setting up the price of items, determining the seller's profit, and making choices for multiobjective real-world functions, are resolved by using TP. Hitchcock [1] introduced the procedure of delivering a product from several sources to several places. After that, the complexity of the TP increased. Maity and Roy [2] developed the multiobjective TP (MOTP). In MOTPs, multiple objectives need to be optimized at the same time, such as minimizing TC, minimizing delivery time, or maximizing customer satisfaction. MOTP is more complex than the single-objective TP. Thereafter, by seeing the uncertainties in real-life problems, in order to vague information, Zadeh [3] developed the concept of fuzzy sets (FSs) to deal with vague information, which has been extensively applied in many areas. It is the extension of the classical set. Atanassov [4] introduced the model of intuitionistic FS (IFS), inspired by the idea of the FS. It is denoted by the grade of MD and NMD; also, it is restricted so that the total of these two degrees cannot exceed one. Nonetheless, in certain authentic-world scenarios, the total of MD and NMD might exceed unity; however, the total of their squares can never exceed unity. As an extension of IFS, Torra [5] gave a definition of hesitant FS (HFS) corresponding to the envelope. In this, they are also introduced to the hesitancy degree with MD and NMD. Thereafter, Xu and Xia et al. [6] developed a dual HFS (DHFS). There are essentially two components to a DHFS: a hesitation membership function that gives the first portion and a hesitancy nonmembership function that gives the second.

In this, the DHFE is denoted in pair$\{(\tilde{h}(p), \tilde{g}(p)$, which also satisfied the circumstances,$0 \leq \tilde{h}_{\breve{D},} \tilde{g}_{\breve{D}} \leq 1, 0 \leq \tilde{h}_{\breve{D}} \widetilde{+g}_{\breve{D}} \leq 1$, and here, $\tilde{h}_{\breve{D}} \in \tilde{h}(p)$, $\tilde{g}_{\breve{D}} \in \tilde{g}(p)$.

Mostly seeing the uncertainty in TP, Chanas [7] did the hybridization of TP and FS and solved a TP under fuzzy parameters. Singh [8] introduced the solution of DHFTP.

In the current situation, most researchers have solved the problem by using the software. However, sometimes we recognize that the problem's equivalent solution can also be found by using traditional methods without the need of mathematical tools, and it is more effective in comparison to the solution obtained through software. According to this study, to get the unique solution of the dual hesitant fuzzy (DHF) MOTP (DHFMOTP) model under consideration, an algorithm is presented, and after this, the comparison of the solutions is shown, in which one is obtained by the existing method and the other is computed by the proposed algorithm. Table 1 shows some recent contributions to the related study.

This research article's primary contributions are as follows:

- Constructing a novel (new) type of TP, called DHFMOTP.
- Introducing a novel score function to defuzzify the DHFMOTP.
- Developing an algorithm for solving DHFMOTP.
- Validating the score function and algorithm by comparing the solution with the existing algorithmic solution.

This work is organized into eight sections. In Section 2, we introduce key concepts essential for understanding this study and basic definitions of HFS, DHFS, and some operations of HFS and DHFS. In Section 3, we present the proposed score function and its properties. The mathematical formulations of MOTP, DHFTP, and DHFMOTP and the proposed methodology are represented in Section 4. In Section 5, numerical computations are discussed. Thereafter, Section 6 deals with the results and discussion related to numerical computations. Section 7 concludes our novel methodology and provides a view for further study. Finally, in the nomenclature list, all abbreviations are mentioned.

1.1. Motivation of This Study. The motivation behind this study stems from the limitations of existing score functions in handling DHF environments within multiobjective transportation models. Traditional methods often assume constant conditions across various scenarios, which oversimplify the complexities present in real-world transportation systems such as fluctuating demand, variable supply, and uncertainty in cost or time. Also, the DHFSs are mostly used in TP because it allows for multiple membership and nonmembership values, which effectively models uncertainty and hesitation in expert judgments. In intricate, real-world transportation circumstances, this results in more precise and adaptable decision-making. Before the research, here arise some questions for the research to find the answers:

Question 1: A traditional score function fails to yield zero despite differences. So, for addressing this demerit, can we develop a score function for improving the discrimination capability between DHF elements?

Answer: In this study, the proposed score function is

$$K(\tilde{d}) = \left\{\frac{2}{M(M+1)}\left|\sum_{i=1}^{M} i\tilde{h}_{\tilde{d}}(p) - \sum_{i=1}^{M} \tilde{g}_{\tilde{d}}(p)\right|. \quad (1)$$

Given the nonzero values for the two distinct DHF numbers (DHFNs), it shows that the proposed score function improves the discrimination capability of DHFN.

Question 2: Can a score function be developed to provide a better evaluation metric compared to the existing score function in DHF environments?

Answer**:** In this study, for addressing the research question, a novel score function is developed, which provide us lower and more consistent score values (e.g., (3.75, 2.83, 1.75) and (1.39, 1.49, 0.66)) compared to the existing function (e.g., (14.5, 14.01, 11.66)), indicating improved optimization performance and greater sensitivity to fuzzy hesitation in both Problem 1 and Problem 2.

Question 3: Can we develop a methodology which is more robust across different problem instances than the existing methodology?

Answer: In this paper, to address this research query, a methodology is developed which delivers consistent results across both methodologies and problem instances to demonstrate the robustness and reliability under varying model configurations.

1.2. Related Works. Hitchcock [1] introduced a novel concept for the dissemination of a commodity from various sources to multiple places. Thereafter, Koopmans [17] provided an application of the optimal resource allocation theory to a specific industry; as a result, I will not be speaking about the theory generally. There are many uncertainties in real life for dealing with this ambiguity; Zadeh [3] created FS. In addition, these kinds of sets are shaped and expanded by the ideas of complement, incorporation, relation, intersection, convexity, union, etc. In the context of FS, many highlights of these ideas are developed. For dealing with unclear demand and unclear supply, Chanas

Table 1: Noteworthy recent works related to this work.

Authors	Type of TP	Hesitant fuzzy set	Dual hesitant fuzzy set
Kumar et al. [9]	FTP	No	Yes
Adhami and Ahmad [10]	MOTP	Yes	No
Saranya and Vinotha [11]	Multiobjective fraction TP	No	Yes
Ahmad et al. [12]	MOTP	Yes	No
Santos-Arteaga et al. [13]	Sustainable TP	Yes	No
Ghosh [14]	Sustainable multiobjective solid TP	Yes	No
Farnam and Darehmiraki [15]	Supply chain management problem	Yes	No
Sharma et al. [16]	Green transportation problem	Yes	No
Proposed method	MOTP	Yes	Yes

et al. [7] applied FS in TPs. As an extension of FS, in [4], Atanassov introduced IFS. In this article, Atanassov introduced IFS from both the MD and NMD of a set. As an extension of IFS, Torra [5] developed HFS and also defined the operations of HFS and the relationship with IFS. Chen and Lee [18] introduced likelihood-based comparison relations for Type 1 and Type 2 IVFS, presented a new ranking method, and introduced a new fuzzy DM method based on these concepts. Xu and Xia [6] presented HFS, and in this, corresponding similarity measures can be derived from a range of distance measurements. In addition, it investigates the connections among the previously described distance measures and creates several additional hesitant ordered weighted similarity and hesitant ordered weighted distance measures. After this, Xu and Xia [6] introduced DHFSs, which are special instances of fuzzy multisets, IFS, FS, HFS, and also looked into the fundamental features and functions of DHFSs. Chen et al. [19] introduced a new method for MCDM using IVIFS, focusing on ranking these fuzzy values and evaluating decision-makers' alternatives. Mahapatra et al. [20] focused on a multichoice stochastic TP, in which the constraints provide and order both factors that have an extreme value distribution. Singh [8] introduced a new similarity measure between DHFS, considering MD and NMD, and demonstrated the ability to obtain corresponding distance measures. Kundu et al. [21] provided an analysis that was conducted on two fixed-charge TP with fuzzy Type 2 parameters. Unit TCs, supply, and demands in the second problem and unit TC and fixed costs in the first problem are examples of Type 2 fuzzy variables. Ye [22] introduced an enhanced cross-entropy measure for single-valued NSs and extend it to interval NSs, utilizing it for MCDM with single-valued and interval neutrosophic information. Chen et al. [23] introduced a new fuzzy MCDM strategy that utilizes transformation methods between IF values and right-angled triangular fuzzy numbers in addition to the proposed IF geometric averaging operators. Maity and Roy [2, 24] created a mathematical model for a TP possessing a multichoice demand, nonlinear cost, and multiobjective conditions. The suggested TP has noncommensurable objective functions that contradict one another. This paper examines an MOTP in an uncertain environment, incorporating reliability into TC and justifying its effectiveness. Rani and Gulati [25] examined a complete fuzzy multiobjective multi-item solid TP and introduced a fuzzy optimal-compromise solution finding method based on fuzzy programming. Roy et al. [26] examined the MOTP in an IF environment, considering market fluctuations and imperfect supply, demand, and TCs characteristics. Kaur et al. [27] suggested a quick and easy way to find the linear MOTP's best compromise option, which is preferred by decision-makers. Maity et al. [28] using DHFN introduced the idea of ambiguity in a TP. In order to create a mathematical model, we consider the DHFN and the decision-maker's ability to deliver the products. Tang et al. [29] explored DHPF Heronian mean operators, revealing their precision compared to current techniques. They also explored their significant characteristics and created a MADM model using DHPFNs. Kumar et al. [9] discussed the inappropriateness of an existing approach for DHFTP, highlighting that it is unable to identify the best option and generates different optimal TCs. A new Mehar score function is proposed to address this issue. Adhami and Ahmad [10] presented a new Pythagorean hesitant fuzzy (PHF) computational algorithm for addressing a multiobjective TP with fuzzy parameters, utilizing potential values for MD and NMD. Nishad and Abhishek [30] introduced a new ranking method for IFN using distance minimizer and MD and NMD functions. It solves TPs involving cost, supplies, and requirements in terms of IFN. The method outperforms existing methods, as demonstrated by an example. Sharma et al. [31] proposed a soft computing optimization technique for solving a multiobjective aspirational level fractional TP, creating a mathematical model based on the highest objective value. Saranya and Vinotha [11] addressed a multiobjective fractional TP using DHFN. Traditional FS provide single membership values, but HFS offer multiple membership degrees. Then, a new method is proposed to solve this problem with nonlinear discount cost, optimizing objective function ratios. The most contentious topic in recent years has been how greenhouse gas emissions, such as those of Co_2 and CH_4, cause air pollution and global warming. This is what inspired Midya et al. [32], who presented a novel solution to the multistage, multiobjective fixed-charge solid TP (MMFSTP) in an intuitionistic fuzzy environment using a green supply chain network system. Fuzzy decision has gained significant attention in science, engineering, economics, and business, particularly in TPs. In such conditions, Ghosh et al. [33] examined a solid TP with fixed charges within all of the data, which are fuzzy integers with intuitionistic membership and nonmembership functions in a multiobjective setting. Ahmad et al. [12] presented a mathematical framework for decision-making, utilizing hesitant fuzzy aggregation operators (AOs). It discussed both linear and hyperbolic membership functions, incorporating hesitant degrees for different objectives. Santos-Arteaga et al. [13] explored the reliability of MCDM evaluations and the influence of experts on rankings, emphasizing the need for strategic incentives in real-life environmental and sustainable strategic problems. Ghosh et al. [14] provided a solid transportation model with many objectives for waste management in the forestry and agriculture departments. The model took into account job opportunities, TCs, and the reduction of carbon emissions through carbon mechanism policies. Niksirat [34] aimed to apply a novel method depending on the nearest interval approximation to identify Pareto optimal solutions for a complete fuzzy MOTP. Zhou et al. [35] optimized the DHFF-TP, one of the suggested ranking functions, which was used to support the proposal of an algorithm. In DHFF environments, artificial neural networks are also used to solve transportation-related issues. Farnam and Darehmiraki [15] discussed the use of hesitant fuzzy linear programming (HFLP) in modeling of multiobjective, three-level supply chain problems. Jabeen et al. [36] using Aczel–Alsina operational principles, AOs, including power Bonferroni mean (PBM) and power average (PA), were designed, and their unique properties were supplied for resolving numerical problems. Sharma et al. [16] introduced a time-sequential probabilistic HFS and a multipurpose green three-dimensional transportation system. Niluminda and Ekanayake [37] presented a novel alternative algorithm for MOTP, addressing the issue where a single objective is inapplicable, utilizing geometric means and the penalty technique to optimize multiple objectives. Bui et al. [38] introduced new concepts for the first time, due to the fact that NF-sets were defined, and theorems were used to demonstrate their characteristics, and an effective MCDM algorithm was recommended. Sharma and Chaudhary [39] introduced the time-sequential DHFS (TS-DHFS), which is a tool for dealing with uncertainty in practical situations. It accurately describes hesitant situations using a time sequence framework. Chaudhary et al. [40] introduced a TS probabilistic Fermatean hesitant set (FHS) (TS-PFHS) and triangular TS-PFHN and proposed an algorithm for a sustainable green transportation model using fuzzy programming and weighted sum technique. Shahzadi et al. [41] discussed the selection of materials in engineering design using (p, q)-ROFS, emphasizing the importance of this crucial phase in ensuring optimal output, profitability, and manufacturer reputation. Ibrahim [42] introduced a complicated n, m-rung orthopair FS (Cn,m-ROFS), a novel tool that integrates quantitative and qualitative studies to handle ambiguity and uncertainty in MADM. Hussain et al. [43] discussed the T-Spherical FS (T-SFS) concepts, a modified version of IFS and their ability to handle vague, unpredictable human opinions, using Heronian mean operators and Aczel–Alsina aggregation tools. Garg [44] proposed an extension of the single-valued neutrosophic set, incorporating exponential and logarithmic parameters to increase their diversity. Ritu et al. [45] developed a novel Pythagorean fuzzy score function to optimize transportation models.

2. Basic definitions

To tackle the uncertainty in TPs using DHFNs, it is essential to first give the fundamental definitions and properties of HFS and DHFS.

2.1. A HFS [5]. $\widetilde{H}$ on a set P is given by the function $\widetilde{h}(p)$, and when this is applied to P, it yields one of its power set components and a subset of values in the interval [0, 1] as

$$\widetilde{h}: P \longrightarrow \psi([0,1]). \tag{2}$$

2.2. A HFS [6]. $\widetilde{H}$ is mathematically defined as follows:

$$\widetilde{H} = \left\{\left(p_i, \widetilde{h}(p_i)\right): p_i \in P\right\}, \tag{3}$$

where $\widetilde{h}(p_i) \in [0,1]$ for each $p_i \in P$, and each member of $\widetilde{h}(p_i)$ is called a HFE, denoted by $\widetilde{h}_i$.

Suppose HFE $(\widetilde{h}_i)$ of HFS $(\widetilde{H})$ for some $p_i \in P$. Then, its infimum and supremum are defined as $h_i^- = \inf_{p_i \in P} \widetilde{h}(p_i)$ and $h_i^+ = \sup_{p_i \in P} \widetilde{h}(p_i)$, respectively.

Note that, in HFS, the membership function assigns a range of values to each element, reflecting uncertainty or ambiguity.

Construction:

1. Define the universe of discourse: Choose the set of elements (P) that you wish to define membership for.
2. Specify the membership range: For each element p in P, identify a set (P) of possible membership values within the interval. This set represents the hesitancy, with each value in the set suggesting a different degree of membership.
3. Express as a function: Represent this mapping from elements to ranges as a function

$$\widetilde{h}: P \longrightarrow \psi([0,1]). \tag{4}$$

Interpretation:

1. Varied membership: The fact that each element is associated with a range of membership values allows for a more nuanced representation of uncertainty compared to traditional FS.
2. Hesitancy: The range reflects the "hesitancy" in assigning a single, definitive MD to each element, recognizing that there may be varying degrees of certainty about its membership.
3. Decision-making: HFS can be valuable in DM scenarios where the information is incomplete or

ambiguous, as it allows for a more flexible representation of uncertainty.

Example 1. Consider a FS "tall people." A traditional FS might assign a membership value of 0.8 to a person who is 6′0″. An HFS could assign a range such as {0.7, 0.8, 0.9}, reflecting the possibility of the person being tall, but with some uncertainty about the exact degree of tallness.

2.3. Score of an HFS. Let us consider an HFS

$$\widetilde{H} = \left\{\left(p_i, \widetilde{h}(p_i)\right): p_i \in P\right\}, \tag{5}$$

where $P = \{p_1, p_2, \ldots, p_n\}$ is a finite set. For a HFE $\widetilde{h}_i$ in $\widetilde{H}$, we define an existing score function, denoted as $\widetilde{S}(h_i)$ and defined as follows:

$$\widetilde{S}(h_i) = \frac{\sum_{j=1}^{n} h(p_j)}{n} \quad (i = 1, 2, 3 \ldots, \ldots, \ldots, m), \tag{6}$$

where $\widetilde{S}(h_i)$ is called the score of h_i .

Let h_1 and h_2 be two HFEs.

Case 1. If $\widetilde{S}(h_1) > \widetilde{S}(h_2)$, then h_1 is greater than h_2.

Case 2. If $\widetilde{S}(h_1) < \widetilde{S}(h_2)$, then h_1 is less than h_2.

Case 3. If $\widetilde{S}(h_1) = \widetilde{S}(h_2)$, then h_1 is equal to h_2.

Let P be a stable set, then a DHFS [8] $\breve{D}$ on P is represented as follows:

$$\breve{D} = \left\{(p, \widetilde{h}(p), \widetilde{g}(p): p \in P\right\}. \tag{7}$$

Here, $\widetilde{h}(p), \widetilde{g}(p) \in [0,1]$, which are, respectively, MD and NMD of any element $p \in P$ to the set $\breve{D}$. Also, we satisfied the conditions,

$$\begin{aligned} &0 \le \widetilde{h}_{\breve{D}}, \widetilde{g}_{\breve{D}} \le 1, \\ &0 \le \widetilde{h}_{\breve{D}} \widetilde{+} g_{\breve{D}} \le 1. \end{aligned} \tag{8}$$

For any $\widetilde{h}_{\breve{D}} \in \widetilde{h}(p)$, $\widetilde{g}_{\breve{D}} \in \widetilde{g}(p)$. DHFE is denoted in pair $\widetilde{d}(p) = (\widetilde{h}(p), \widetilde{g}(p))$.

DHFE satisfies the following criteria:

i. Total uncertainty: $\widetilde{d} = \{(0), (1)\}$.

ii. Total certainty: $\widetilde{d} = \{(1), (0)\}$.

iii. Total ill knowledge: $\widetilde{d} = [0, 1]$.

iv. Nonsensical element: $\widetilde{d} = \varnothing (h = \varnothing, g = \varnothing)$.

2.4. Arithmetic Operations on DHFEs. Let $\widetilde{d}_1 = \left\{\widetilde{h}_{\widetilde{d}_1}, \widetilde{g}_{\widetilde{d}_1}\right\}$ and $\widetilde{d}_2 = \left\{\widetilde{h}_{\widetilde{d}_2}, \widetilde{g}_{\widetilde{d}_2}\right\}$ be any two DHFEs; then add, subtract, and scalar multiply the numbers defined as follows.

Addition:

$$\widetilde{d}_1 \oplus \widetilde{d}_2 = \left\{\widetilde{h}_{\widetilde{d}_1} \oplus \widetilde{h}_{\widetilde{d}_2}, \widetilde{g}_{\widetilde{d}_1} \odot \widetilde{g}_{\widetilde{d}_2}\right\} = \bigcup_{\gamma_{\widetilde{d}_1} \epsilon \widetilde{h}_{\widetilde{d}_1}, \eta_{\widetilde{d}_1} \epsilon \widetilde{g}_{\widetilde{d}_1}, \gamma_{\widetilde{d}_2} \epsilon \widetilde{h}_{\widetilde{d}_2}, \eta_{\widetilde{d}_2} \epsilon \widetilde{g}_{\widetilde{d}_2}} = \left\{\gamma_{\widetilde{d}_1} + \gamma_{\widetilde{d}_2} - \gamma_{\widetilde{d}_1}\gamma_{\widetilde{d}_2}\left(\eta_{\widetilde{d}_1}\eta_{\widetilde{d}_2}\right)\right\}. \tag{9}$$

Subtraction:

$$\widetilde{d}_1 \ominus \widetilde{d}_2 = \left\{\widetilde{h}_{\widetilde{d}_1} \ominus \widetilde{h}_{\widetilde{d}_2}, \widetilde{g}_{\widetilde{d}_1} \ominus \widetilde{g}_{\widetilde{d}_2}\right\} = \bigcup_{\gamma_{\widetilde{d}_1} \epsilon \widetilde{h}_{\widetilde{d}_1}, \eta_{\widetilde{d}_1} \epsilon \widetilde{g}_{\widetilde{d}_1}, \gamma_{\widetilde{d}_2} \epsilon \widetilde{h}_{\widetilde{d}_2}, \eta_{\widetilde{d}_2} \epsilon \widetilde{g}_{\widetilde{d}_2}} = \left\{\gamma_{\widetilde{d}_1}\gamma_{\widetilde{d}_2}, \left(\eta_{\widetilde{d}_1} + \eta_{\widetilde{d}_2} - \eta_{\widetilde{d}_1}\eta_{\widetilde{d}_2}\right)\right\}. \tag{10}$$

Scalar multiplication: for any real number k,

$$k\widetilde{d} = \bigcup_{\gamma_{\widetilde{d}} \epsilon \widetilde{h}_{\widetilde{d}}, \eta_{\widetilde{d}} \epsilon \widetilde{g}_{\widetilde{d}}} = \left\{1 - (1 - \gamma_{\widetilde{d}})^n (\eta_{\widetilde{d}})^n\right\}, \tag{11}$$

if $k \in z^+$, then for any k,

$$\widetilde{d}^k = \bigcup_{\gamma_{\widetilde{d}} \epsilon \widetilde{h}_{\widetilde{d}}, \eta_{\widetilde{d}} \epsilon \widetilde{g}_{\widetilde{d}}} = \left\{(\gamma_{\widetilde{d}})^n, 1 - (1 - \eta_{\widetilde{d}})^n\right\}. \tag{12}$$

2.5. Score Function of DHFS. Let $\breve{D} = (p, \widetilde{h}(p), \widetilde{g}(p): p \in P\}$ be a DHFS, where $P = \{p_1, p_2, \ldots, p_k\}$ and $\widetilde{d} = \left\{\widetilde{h}_{\widetilde{d}}, \widetilde{g}_{\widetilde{d}}\right\}$ be DHFSs, Then, the existing score function $S_{\widetilde{d}}$ is represented as

$$S_{\widetilde{d}} = \left|\frac{1}{n}\sum_{i=1}^{n} \widetilde{h}_{\widetilde{d}}(p) - \frac{1}{n}\sum_{i=1}^{n} \widetilde{g}_{\widetilde{d}}(p)\right|. \tag{13}$$

Let $\widetilde{d}_1$ and $\widetilde{d}_2$ be any two DHFSs. Then, the order relation is as follows:

i. If $S_{\widetilde{d}_1} > S_{\widetilde{d}_2}$, then $\widetilde{d}_1 > \widetilde{d}_2$

ii. If $S_{\widetilde{d}_1} < S_{\widetilde{d}_2}$, then $\widetilde{d}_1 < \widetilde{d}_2$

iii. If $S_{\widetilde{d}_1} = S_{\widetilde{d}_2}$, then $\widetilde{d}_1 = \widetilde{d}_2$

3. A Novel Score Function $(K(\widetilde{d}))$ of DHFS

In this section, we developed a new score function.

Let $\widetilde{d} = \{\widetilde{h}_{\widetilde{d}}, \widetilde{g}_{\widetilde{d}}\}$ be a DHFS, Then, the proposed score function $K(\widetilde{d})$ is defined as

$$K(\widetilde{d}) = \left\{\frac{2}{M(M+1)}\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right|. \quad (14)$$

Here, $K(\widetilde{d}) \in [0, 1]$

3.1. The Proposed Score Function's Methodology.

$$K(\widetilde{d}) = \left\{\frac{2}{M(M+1)}\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right|. \quad (15)$$

Proof 1. To make a novel score function, we studied many papers which are related to the score function of DHFS. We find a score function, Maity et al. [28], $S_{\widetilde{d}} = |(1/n)\sum_{i=1}^{n}\widetilde{h}_{\widetilde{d}}(p) - (1/n)\sum_{i=1}^{n}\widetilde{g}_{\widetilde{d}}(p)|$ of DHFS, and after analyzing this score function, we observe the demerit of this score function as □

For example, let $\widetilde{h}_{\widetilde{d}}(p) = \{0.1, 0.4, 0.7\}$ and $\widetilde{g}_{\widetilde{d}}(p) = \{0.2, 0.6, 0.4\}$; so, here $\widetilde{h}_{\widetilde{d}}(p) \neq \widetilde{g}_{\widetilde{d}}(p)$ and also $\widetilde{h}_{\widetilde{d}}(p) = \widetilde{g}_{\widetilde{d}}(p) \neq 0$ but

$$S_{\widetilde{d}} = \left(\frac{1.2}{3} - \frac{1.2}{3}\right), \qquad S_{\widetilde{d}} = 0. \quad (16)$$

Then, for reducing the demerits of this score function and for achieving our goal, the membership and non-membership coefficients are being experimented with, and their average factor is obtained; then, we develop a novel score function,

$$K(\widetilde{d}) = \left\{\frac{2}{M(M+1)}\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right|, \quad (17)$$

which lies between [0, 1]. This gives us a lower score value than the one that already exists.

For example, let $\widetilde{h}_{\widetilde{d}}(p) = \{0.1, 0.4, 0.7\}$ and $\widetilde{g}_{\widetilde{d}}(p) = \{0.2, 0.6, 0.4\}$; so, here $\widetilde{h}_{\widetilde{d}}(p) \neq \widetilde{g}_{\widetilde{d}}(p)$ and also $\widetilde{h}_{\widetilde{d}}(p) = \widetilde{g}_{\widetilde{d}}(p) \neq 0$, then

$$K(\widetilde{d}) = \frac{2}{12}|3 - 1.2| = \left|\frac{2 * 1.8}{12}\right|, \qquad K(\widetilde{d}) = 0.3 \neq 0. \quad (18)$$

Here, the term $2/M(M+1)$ is a normalization constant. The variable M may stand for the number of unique membership values or categories in the DHFS. The boundedness and comparability of the score across other datasets or contexts are guaranteed by this standardization. The expression $\sum_{i=1}^{n} i\widetilde{h}_{\widetilde{d}}(p)$ denotes the weighted sum of the DHFS values at each index i, where *ith* may represent the index's weight. This is the total of another function $\sum_{i=1}^{M}\widetilde{g}_{\widetilde{d}}(p)$, which is computed by this term over the same indices. The mode ensures that the score is non-negative and reflects the magnitude of deviation.

3.2. The Proposed Score Function $K(\widetilde{d})$, Where $\widetilde{d} = \{\widetilde{h}_{\widetilde{d}}, \widetilde{g}_{\widetilde{d}}\}$ Lies Between [0, 1]

Proof 2. Let, $\widetilde{d} = \{\widetilde{h}_{\widetilde{d}}, \widetilde{g}_{\widetilde{d}}\}$ be a DHFS, Here,

$$0 \leq \widetilde{h}_{\widetilde{d}}(p) \leq 1, \quad (19)$$

$$0 \leq \widetilde{g}_{\widetilde{d}}(p) \leq 1. \quad (20)$$

□

Multiplying by "i" in equation (19), we get

$$0 \leq i\widetilde{h}_{\widetilde{d}}(p) \leq i. \quad (21)$$

Taking summation in equation (21), $0 \leq \sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) \leq \sum_{i=1}^{M} i$

$$0 \leq \sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) \leq \frac{M(M+1)}{2} \text{ (by using the formula sum of first } M \text{ natural numbers).} \quad (22)$$

Now, from equation (20), we have

$$\sum_{i=1}^{M}\widetilde{g}_{\widetilde{d}}(p) \geq 0, \qquad -\sum_{i=1}^{M}\widetilde{g}_{\widetilde{d}}(p) \leq 0. \quad (23)$$

Adding equations (22) and (23) gives

$$\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M}\widetilde{g}_{\widetilde{d}}(p) \leq \frac{M(M+1)}{2} + 0. \quad (24)$$

Taking the mode on both sides, we have

$$\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right| \le \left|\frac{M(M+1)}{2}\right|. \quad (25)$$

We know that $|(M(M+1)/2)| = M(M+1)/2$, so

$$\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right| \le \frac{M(M+1)}{2},$$
$$\frac{2}{M(M+1)}\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right| \le 1. \quad (26)$$

Obviously,

$$\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right| \ge 0,$$
$$\frac{2}{M(M+1)} \ge 0. \quad (27)$$

Now, multiplying both the equations, we get

$$\frac{2}{M(M+1)}\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right| \ge 0. \quad (28)$$

From equations (26) and (28), we have

$$0 \le \frac{2}{M(M+1)}\left|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{d}}(p) - \sum_{i=1}^{M} \widetilde{g}_{\widetilde{d}}(p)\right| \le 1. \quad (29)$$

Thus,

$$0 \le K(\widetilde{d}) \le 1, \quad \text{where } \widetilde{d} = \left\{\widetilde{h}_{\widetilde{d}}, \widetilde{g}_{\widetilde{d}}\right\}. \quad (30)$$

3.3. Ranking of DHFS. Let us consider two DHFS, $\widetilde{d}_1 = \left\{\widetilde{h}_{\widetilde{d}_1}, \widetilde{g}_{\widetilde{d}_1}\right\}$ and $\widetilde{d}_2 = \left\{\widetilde{h}_{\widetilde{d}_2}, \widetilde{g}_{\widetilde{d}_2}\right\}$, then ranking laws are described in the following way:

- $K(\widetilde{d}_1) > K(\widetilde{d}_2)$ if $f\ \widetilde{d}_1$ is greater then $\widetilde{d}_2$
- $K(\widetilde{d}_1) < K(\widetilde{d}_2)$ if $f\ \widetilde{d}_1$ is less then $\widetilde{d}_2$
- $K(\widetilde{d}_1) = K(\widetilde{d}_2)$ if $f\ \widetilde{d}_1$ is equal to $\widetilde{d}_2$

4. Mathematical Formulation of Existing and Proposed Models and Algorithms

In this part, mathematical models for MOTP, DHFTP, and DHFMOTP are developed.

4.1. MOTP. The mathematical structure of a MOTP is given as follows:

$$\text{optimize } Z^k = \sum_{i=1}^{u}\sum_{j=1}^{v} \widetilde{c}_{ij}^{k}\widetilde{x}_{ij}, \quad (31)$$

$$\text{subject to, } \sum_{j=1}^{v} \widetilde{x}_{ij} = \widetilde{a}_i \quad (i = 1, 2, 3, \ldots, u), \quad (32)$$

$$\sum_{i=1}^{u} \widetilde{x}_{ij} = \widetilde{b}_j \quad (j = 1, 2, 3, \ldots, v), \quad (33)$$

$$\widetilde{x}_{ij} \ge 0, \quad \forall i, j, \quad (34)$$

where $\sum_{i=1}^{u}\sum_{j=1}^{v}\widetilde{c}_{ij}^{k}$ = total TC $\forall\, i = 1, 2, \ldots, u$ & $j = 1, 2, \ldots, v$, $\sum_{i=1}^{u} \widetilde{a}_i$ = total supply, and $\sum_{j=1}^{v}\widetilde{b}_j$ = total demand.

4.2. Type of TP

- Balanced TP:
  Total supply = total demand, i.e.,

$$\sum_{i=1}^{u} \widetilde{a}_i = \sum_{j=1}^{v} \widetilde{b}_j. \quad (35)$$

- Unbalanced TP:
  Total supply ≠ total demand, i.e.,

$$\sum_{i=1}^{u} \widetilde{a}_i \ne \sum_{j=1}^{v} \widetilde{b}_j. \quad (36)$$

- $\sum_{i=1}^{u}\widetilde{a}_i > \sum_{j=1}^{v}\widetilde{b}_j$; here, we need to add a dummy destination in the matrix with zero cost elements.
- $\sum_{i=1}^{u}\widetilde{a}_i < \sum_{j=1}^{v}\widetilde{b}_j$; here, we need to add a dummy source that is included in the matrix with zero cost elements.

4.3. DHFTP [28]. Let us assume DHF costs $(\widetilde{h}_{ij}, \widetilde{g}_{ij})$ for both membership and nonmembership values, where $\widetilde{h}_{ij}$ and $\widetilde{g}_{ij}$ represent the membership and nonmembership values, respectively. To discover the best solution to the optimization problem in this case, we must raise the membership value $\widetilde{h}_{ij}$ and lower the nonmembership value $\widetilde{g}_{ij}$. The following is the optimal solution to the optimization problem:

$$\text{optimize } \widetilde{z} = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\widetilde{h}_{ij}, \widetilde{g}_{ij}\right)\widetilde{x}_{ij}, \quad (37)$$

$$\sum_{j=1}^{v} \widetilde{x}_{ij} \ge \widetilde{a}_i \quad (i = 1, 2, 3, \ldots, u), \quad (38)$$

$$\sum_{i=1}^{u} \widetilde{x}_{ij} \ge \widetilde{b}_j \quad (j = 1, 2, 3, \ldots, v), \quad (39)$$

$$\widetilde{x}_{ij} \ge 0, \quad \forall i, j. \quad (40)$$

Here, we maximize the MD $\widetilde{h}_{ij}$ and minimize the NMD $\widetilde{g}_{ij}$ according to the problem. The objective function of the problem can be reduced into two objective functions as defined in the equation. They are described in the biobjective problem formulation that follows:

$$\text{maximize } \widetilde{z} = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\widetilde{h}_{ij}\right)\widetilde{x}_{ij},$$
$$\text{minimize } \widetilde{z} = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\widetilde{g}_{ij}\right)\widetilde{x}_{ij}, \quad (41)$$

and subject to constraint,

$$\sum_{j=1}^{v} \tilde{x}_{ij} \geq \tilde{a}_i \quad (i = 1, 2, 3, \ldots, u), \tag{42}$$

$$\sum_{i=1}^{u} \tilde{x}_{ij} \geq \tilde{b}_j \quad (j = 1, 2, 3, \ldots, v), \tag{43}$$

$$\tilde{x}_{ij} \geq 0, \quad \forall i, j. \tag{44}$$

Here,0 $\leq \tilde{h}_{ij} \leq 1$ and $0 \leq \tilde{g}_{ij} \leq 1$. The DHFTP is now a LPP with two objective functions because of these model limitations. It is shown as follows:

$$\text{maximize}\, \tilde{z} = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\tilde{h}_{ij}\right)\tilde{x}_{ij}, \tag{45}$$

$$\text{minimize}\, \tilde{z} = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\tilde{g}_{ij}\right)\tilde{x}_{ij}. \tag{46}$$

Subject to

$$\left(\tilde{h}_{ij} + \tilde{g}_{ij} - 1\right)\tilde{x}_{ij} \leq 0, \quad \forall (i = 1, 2, 3, \ldots, u; j = 1, 2, 3, \ldots, n), \tag{47}$$

and subject to constraint,

$$\sum_{j=1}^{v} \tilde{x}_{ij} \geq \tilde{a}_i \quad (i = 1, 2, 3, \ldots, u), \tag{48}$$

$$\sum_{i=1}^{u} \tilde{x}_{ij} \geq \tilde{b}_j \quad (j = 1, 2, 3, \ldots, v), \tag{49}$$

$$\tilde{x}_{ij} \geq 0, \quad \forall i, j. \tag{50}$$

4.4. Proposed Model of DHFMOTP. Let us assume $[(\tilde{h}_{ij})_{k,} (\tilde{g}_{ij})_k]$ as the membership and nonmembership values of the DHFMOTP cost, where the membership value is $(\tilde{h}_{ij})_k$ and the nonmembership value is $(\tilde{g}_{ij})_k$. In this case, the best way to solve the optimization problem is to maximize the membership value $(\tilde{h}_{ij})_k$ and minimize the nonmembership value $(\tilde{g}_{ij})_k$. The following is the optimal solution to the optimization problem:

$$\text{optimize}\left(\tilde{z}_k\right) = \sum_{i=1}^{u}\sum_{j=1}^{v}\left[\left(\tilde{h}_{ij}\right)_{k,}\left(\tilde{g}_{ij}\right)_k\right]\tilde{x}_{ij}. \tag{51}$$

Subject to constraint,

$$\sum_{j=1}^{v} \tilde{x}_{ij} \geq \tilde{a}_i \quad (i = 1, 2, 3, \ldots, u), \tag{52}$$

$$\sum_{i=1}^{u} \tilde{x}_{ij} \geq \tilde{b}_j \quad (j = 1, 2, 3, \ldots, v), \tag{53}$$

$$\tilde{x}_{ij} \geq 0, \quad \forall i, j. \tag{54}$$

Here, we maximize the MD $(\tilde{h}_{ij})_k$ and minimize the NMD$(\tilde{g}_{ij})_k$ according to problem (37). As a result, our problem's objective function is reduced to two. The bio-bjective problem formulation that follows defines them as

$$\begin{aligned} &\text{maximize}\, \tilde{z}_k = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\tilde{h}_{ij}\right)_k\tilde{x}_{ij}, \\ &\text{minimize}\, \tilde{z}_k = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\tilde{g}_{ij}\right)_k\tilde{x}_{ij}, \end{aligned} \tag{55}$$

and subject to constraint,

$$\sum_{j=1}^{v} \tilde{x}_{ij} \geq \tilde{a}_i \quad (i = 1, 2, 3, \ldots, u), \tag{56}$$

$$\sum_{i=1}^{u} \tilde{x}_{ij} \geq \tilde{b}_j \quad (j = 1, 2, 3, \ldots, v), \tag{57}$$

$$\tilde{x}_{ij} \geq 0, \quad \forall i, j. \tag{58}$$

Here, $\sum_{i=1}^{u}\tilde{a}_i$ = total supply, $\sum_{j=1}^{v}\tilde{b}_j$ = total demand and $0 \leq (\tilde{h}_{ij})_k, (\tilde{g}_{ij})_k \leq 1$; these are the constraints of this model; therefore, the DHFMOTP turns into a LPP with two desired objective functions represented as follows:

$$\text{maximize}\, \tilde{z}_k = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\tilde{h}_{ij}\right)_k\tilde{x}_{ij}, \tag{59}$$

$$\text{minimize}\, \tilde{z}_k = \sum_{i=1}^{u}\sum_{j=1}^{v}\left(\tilde{g}_{ij}\right)_k\tilde{x}_{ij}. \tag{60}$$

Subject to

$$\left[\left(\tilde{h}_{ij}\right)_k + \left(\tilde{g}_{ij}\right)_k - 1\right]\tilde{x}_{ij} \leq 0, \tag{61}$$

where $(\tilde{h}_{ij})_k, (\tilde{g}_{ij})_k$ are the membership and nonmembership degrees, respectively, $\forall (i = 1, 2, 3, \ldots, u; j = 1, 2, 3, \ldots, v)$, and subject to constraint,

$$\sum_{j=1}^{v} \tilde{x}_{ij} \geq \tilde{a}_i \quad (i = 1, 2, 3, \ldots, u), \tag{62}$$

$$\sum_{i=1}^{u} \tilde{x}_{ij} \geq \tilde{b}_j \quad (j = 1, 2, 3, \ldots, v), \tag{63}$$

$$\tilde{x}_{ij} \geq 0, \quad \forall i, j. \tag{64}$$

4.5. Modified Proposed Methodology. In this section, the proposed algorithm is introduced, and it is specially designed to address the MOTP within the DHF environment. The proposed algorithm in steps is listed as follows:

Step 1: First, consider the DHFMOTP.

Step 2: Now, use both the existing and proposed score functions and perform defuzzification to obtain the score values.

Step 3: To maintain the equilibrium of MOTP, we perform

$$\sum_{i=1}^{u} \widetilde{a}_i = \sum_{j=1}^{v} \widetilde{b}_j, \tag{65}$$

i.e.,

Requirement = demand.

Add a dummy row or column on demand/requirement if demand ≠ requirement. This will turn the off-balanced MOTP into an equilibrium MOTP, which can then be solved using the necessary procedure.

Step 4: After Step-3, we apply the existing and proposed algorithm and find the geometric mean [14] and arithmetic mean of the score values for each and every row and column.

Step 5: Find the penalty of each row and column by using the formula, given as follows:

$$\left| \left(\widetilde{c}_{ij} \right)_{\max} - \left(\widetilde{c}_{ij} \right)_{\min 2} \right|. \tag{66}$$

Step 6: Select the maximum penalty for each row and column and assign the min (supply, demand) to min $(\widetilde{c}_{ij})$ value cell.

Step 7: If there is a tie in the maximum penalty for a column and row, then choose the min. $(\widetilde{c}_{ij})$ corresponding to the penalty values.

Step 8: Once the allocation in the preceding row matches the supply at the starting point, mark off the relevant row. If it meets the demand there, mark off the corresponding column.

Step 9: The process should stop if the demand is meet at every destination and there is sufficient supply at every origin. If not, go through the previous steps again.

Step 10: Using an MOTP allocation table, find the appropriate effective cost values for every objective.

Figure 1 shows the architecture of our proposed methodology. Note: dual hesitant fuzzy (DHF) environment handling, a unique combination of preexisting and newly proposed score functions for defuzzification, and the application of a modified penalty-based allocation strategy designed especially for MOTP set the suggested algorithm apart from other multiobjective optimization techniques. Our strategy, in contrast to conventional techniques, effectively balances supply and demand by using dummy adjustments and computing allocations using the geometric and arithmetic means of fuzzy score values. This results in more precise and useful solutions for both balanced and unbalanced MOTP.

5. Numerical Computations

To explain the robustness of our developed score function and methodology, we will use some real-life numerical problems. Step 1: Consider the DHFMOTP

Step 1: Consider the DHFMOTP.

Step 2: Determine the score values for each IVPFT cost using the proposed and existing score function, and then replace all of them with that value to obtain a clearly defined DHFMOTP. In Tables 3 and 4, the defuzzified DHFMOTP is shown.

Step 3: Now, we follow the methodology (Section 4.5) as mentioned in the proposed algorithm. Following this methodology, we get the optimal results, which are given in Table 5.

Problem 1. A logistics company operates two warehouses (S_1 and S_2) to supply goods to two retail outlets (D_1 and D_2). The company aims to make optimal transportation decisions considering the three conflicting objectives: TC (per ton), reduce monetary expenditure; transportation time (TT) (hours/days), ensure timely deliveries; environmental impact (EI) (kg/ton), minimize emissions and fuel usage. Due to uncertainty and decision-maker hesitation in evaluating the performance of each transport route, all objective evaluations are expressed using DHFNs, which allow for multiple possible values of both membership and nonmembership. The tabular representation of the DHFMOTP is presented in Table 2.

$$\begin{aligned}
&\left\{\left(\widetilde{h}_{11}\right)_{1,}\left(\widetilde{g}_{11}\right)_{1}\right\} = \{(0.2, 0.3)\,(0.8, 0.6)\} \quad \left\{\left(\widetilde{h}_{11}\right)_{2,}\left(\widetilde{g}_{11}\right)_{2}\right\} = \{(0.1, 0.2)\,(0.7, 0.4)\},\\
&\left\{\left(\widetilde{h}_{11}\right)_{3,}\left(\widetilde{g}_{11}\right)_{3}\right\} = \{(0.3, 0.1)\,(0.5, 0.1)\} \quad \left\{\left(\widetilde{h}_{12}\right)_{1,}\left(\widetilde{g}_{12}\right)_{1}\right\} = \{(0.4, 0.1)\,(0.3, 0.6)\},\\
&\left\{\left(\widetilde{h}_{12}\right)_{2,}\left(\widetilde{g}_{12}\right)_{2}\right\} = \{(0.2, 0.3)\,(0.6, 0.4)\} \quad \left\{\left(\widetilde{h}_{12}\right)_{3,}\left(\widetilde{g}_{12}\right)_{3}\right\} = \{(0.3, 0.1)\,(0.6, 0.5)\},\\
&\left\{\left(\widetilde{h}_{21}\right)_{1,}\left(\widetilde{g}_{21}\right)_{1}\right\} = \{(0.1, 0.2)\,(0.9, 0.6)\} \quad \left\{\left(\widetilde{h}_{21}\right)_{2,}\left(\widetilde{g}_{21}\right)_{2}\right\} = \{(0.3, 0.1)\,(0.7, 0.5)\},\\
&\left\{\left(\widetilde{h}_{21}\right)_{3,}\left(\widetilde{g}_{21}\right)_{3}\right\} = \{(0.4, 0.3)\,(0.5, 0.4)\} \quad \left\{\left(\widetilde{h}_{22}\right)_{1,}\left(\widetilde{g}_{22}\right)_{1}\right\} = \{(0.2, 0.5)\,(0.8, 0.3)\},\\
&\left\{\left(\widetilde{h}_{22}\right)_{2,}\left(\widetilde{g}_{22}\right)_{2}\right\} = \{(0.3, 0.2)\,(0.5, 0.6)\} \quad \left\{\left(\widetilde{h}_{22}\right)_{3,}\left(\widetilde{g}_{22}\right)_{3}\right\} = \{(0.1, 0.3)\,(0.9.0.5)\}.
\end{aligned} \tag{67}$$

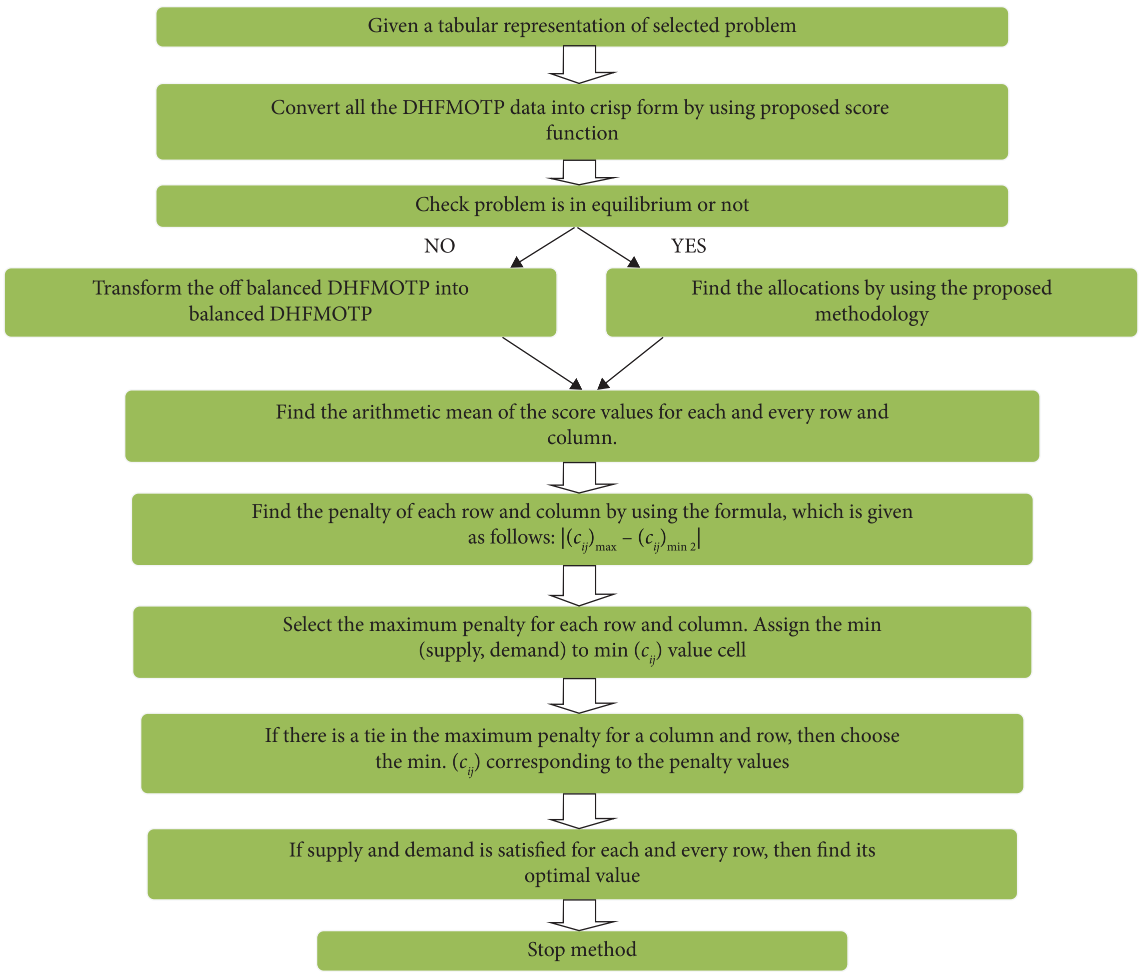

Figure 1: Architecture of the proposed algorithm.

Table 2: Dual hesitant fuzzy multiobjective transportation problem.

	D_1	D_2	**Supply** $(\tilde{a}_i)$
S_1	$\{(\tilde{h}_{11})_1,(\tilde{g}_{11})_1\}$ $\{(\tilde{h}_{11})_2,(\tilde{g}_{11})_2\}$ $\{(\tilde{h}_{11})_3,(\tilde{g}_{11})_3\}$	$\{(\tilde{h}_{12})_1,(\tilde{g}_{12})_1\}$ $\{(\tilde{h}_{12})_2,(\tilde{g}_{12})_2\}$ $\{(\tilde{h}_{12})_3,(\tilde{g}_{12})_3\}$	**5**
S_2	$\{(\tilde{h}_{21})_1,(\tilde{g}_{21})_1\}$ $\{(\tilde{h}_{21})_2,(\tilde{g}_{21})_2\}$ $\{(\tilde{h}_{21})_3,(\tilde{g}_{21})_3\}$	$(\tilde{h}_{22})_1,(\tilde{g}_{22})_1\}$ $\{(\tilde{h}_{22})_2,(\tilde{g}_{22})_2\}$ $\{(\tilde{h}_{22})_3,(\tilde{g}_{22})_3\}$	**3**
Demand $(\tilde{b}_j)$	**6**	**2**	

Note: The bold values represent the crisp supply and demand quantities that ensure feasibility of the transportation problem.

Problem 2. After a severe natural disaster (e.g., a flood in eastern India), a humanitarian logistics organization (such as the Red Cross or NDMA) needs to distribute emergency supplies (food, medicine, water) from its main supply depots to affected cities. The objective is to perform this in a way that is cost-effective, quick, and environmentally conscious (suppliers):

S_1-New Delhi Relief Depot (capacity: 20 tons),

S_2-Kolkata Depot (capacity: 25 tons),

Table 3: Crisp form of Dual hesitant fuzzy multiobjective transportation Problem 1 by existing score function.

	D_1	D_2	Supply $(\tilde{a}_i)$
S_1	0.45 0.4 0.1	0.20 0.25 0.35	5
S_2	0.6 0.4 0.1	0.25 0.30 0.5	3
Demand $(\tilde{b}_j)$	**6**	**2**	

Note: The bold values represent the crisp supply and demand quantities that ensure feasibility of the transportation problem.

S_3-Mumbai Depot (capacity: 30 tons). Affected cities (destinations):

D_1-Guwahati (requires 25tons),

D_2-Patna (requires 20 tons),

D_3-Ranchi (requires 30 tons).

Objectives (multiobjective): Total TC, monetary cost (per ton); TT, critical for life-saving operations (hours/days); and EI, CO_2 emissions (kg/ton) depending on vehicle type and route. The tabular representation of the DHFMOTP is presented in Table 6.

Here,

Step 1: Consider the DHFMOTP.

$$\begin{aligned}
&\left\{\left(\tilde{h}_{11}\right)_{1,}\left(\tilde{g}_{11}\right)_{1}\right\} = \{(0.5.0.4, 0.1)(0.4, 0.5, 0.9)\}, \left\{\left(\tilde{h}_{11}\right)_{2,}\left(\tilde{g}_{11}\right)_{2}\right\} = \{(0.4, 0.2, 0.3)(0.1, 0.8, 0.4)\},\\
&\left\{\left(\tilde{h}_{11}\right)_{3,}\left(\tilde{g}_{11}\right)_{3}\right\} = \{(0.1, 0.2, 0.4)(0.6, 0.3, 0.2)\}, \left\{\left(\tilde{h}_{12}\right)_{1,}\left(\tilde{g}_{12}\right)_{1}\right\} = \{(0.3, 0.2, 0.1)(0.5, 0.3, 0.4)\},\\
&\left\{\left(\tilde{h}_{12}\right)_{2,}\left(\tilde{g}_{12}\right)_{2}\right\} = \{(0.4, 0.3, 0.1)(0.4, 0.6, 0.2)\}, \left\{\left(\tilde{h}_{12}\right)_{3,}\left(\tilde{g}_{12}\right)_{3}\right\} = \{(0.3, 0.1, 0.4)(0.4, 0.5, 0.2)\},\\
&\left\{\left(\tilde{h}_{13}\right)_{1,}\left(\tilde{g}_{13}\right)_{1}\right\} = \{(0.6, 0.2, 0.1)(0.3, 0.6, 0.8)\}, \left\{\left(\tilde{h}_{13}\right)_{2,}\left(\tilde{g}_{13}\right)_{2}\right\} = \{(0.3, 0.2, 0.4)(0.7, 0.4, 0.5)\},\\
&\left\{\left(\tilde{h}_{13}\right)_{3,}\left(\tilde{g}_{13}\right)_{3}\right\} = \{(0.2, 0.4, 0.1)(0.4, 0.2, 0.8)\}, \left\{\left(\tilde{h}_{21}\right)_{1,}\left(\tilde{g}_{21}\right)_{1}\right\} = \{(0.3, 0.5, 0.1)(0.6, 0.4, 0.2)\},\\
&\left\{\left(\tilde{h}_{21}\right)_{2,}\left(\tilde{g}_{21}\right)_{2}\right\} = \{(0.2, 0.5, 0.1)(0.7, 0.3, 0.2)\}, \left\{\left(\tilde{h}_{21}\right)_{3,}\left(\tilde{g}_{21}\right)_{3}\right\} = \{(0.5, 0.2, 0.1)(0.1, 0.2, 0.8)\},\\
&\left\{\left(\tilde{h}_{22}\right)_{1,}\left(\tilde{g}_{22}\right)_{1}\right\} = \{(0.5, 0.3, 0.1)(0.2, 0.3, 0.7)\}, \left\{\left(\tilde{h}_{22}\right)_{2,}\left(\tilde{g}_{22}\right)_{2}\right\} = \{(0.3, 0.4, 0.2)(0.6, 0.5, 0.7)\},\\
&\left\{\left(\tilde{h}_{22}\right)_{3,}\left(\tilde{g}_{22}\right)_{3}\right\} = \{(0.2, 0.6, 0.1)(0.5, 0.2, 0.7)\}, \left\{\left(\tilde{h}_{23}\right)_{1,}\left(\tilde{g}_{23}\right)_{1}\right\} = \{(0.5, 0.3, 0.2)(0.3, 0.4, 0.7)\},\\
&\left\{\left(\tilde{h}_{23}\right)_{2,}\left(\tilde{g}_{23}\right)_{2}\right\} = \{(0.6, 0.2, 0.1)(0.4, 0.7, 0.8)\}, \left\{\left(\tilde{h}_{23}\right)_{3,}\left(\tilde{g}_{23}\right)_{3}\right\} = \{(0.6, 0.1, 0.2)(0.2, 0.3, 0.7)\},\\
&\left\{\left(\tilde{h}_{31}\right)_{1,}\left(\tilde{g}_{31}\right)_{1}\right\} = \{(0.3, 0.2, 0.1)(0.2, 0.6, 0.7)\}, \left\{\left(\tilde{h}_{31}\right)_{2,}\left(\tilde{g}_{31}\right)_{2}\right\} = \{(0.5, 0.3, 0.2)(0.4, 0.6, 0.8)\},\\
&\left\{\left(\tilde{h}_{31}\right)_{3,}\left(\tilde{g}_{31}\right)_{3}\right\} = \{(0.2, 0.1, 0.4)(0.7, 0.5, 0.6)\}, \left\{\left(\tilde{h}_{32}\right)_{1,}\left(\tilde{g}_{32}\right)_{1}\right\} = \{(0.4, 0.2, 0.3)(0.5, 0.6, 0.4)\},\\
&\left\{\left(\tilde{h}_{32}\right)_{2,}\left(\tilde{g}_{32}\right)_{2}\right\} = \{(0.3, 0.1, 0.2)(0.2, 0.7, 0.4)\}, \left\{\left(\tilde{h}_{32}\right)_{3,}\left(\tilde{g}_{32}\right)_{3}\right\} = \{(0.1, 0.2, 0.4)(0.8, 0.3, 0.2)\},\\
&\left\{\left(\tilde{h}_{33}\right)_{1,}\left(\tilde{g}_{33}\right)_{1}\right\} = \{(0.4, 0.1, 0.2)(0.5, 0.7, 0.4)\}, \left\{\left(\tilde{h}_{33}\right)_{2,}\left(\tilde{g}_{33}\right)_{2}\right\} = \{(0.5, 0.2, 0.1)(0.2, 0.8, 0.7)\},\\
&\left\{\left(\tilde{h}_{33}\right)_{3,}\left(\tilde{g}_{33}\right)_{3}\right\} = \{(0.1, 0.4, 0.3)(0.6, 0.3, 0.5)\}.
\end{aligned} \tag{68}$$

Step 2: Determine the score value by using the existing and proposed score functions. We have a clearly defined DHFMOTP once we substitute the value for each IVPFT cost. Tables 7 and 8 display the defuzzified DHFMOTP.

Step 3: Now, we follow the necessary methodology (Section 4.5) as mentioned in the proposed algorithm.

Following this methodology, we get optimal results, which are given in Table 9.

6. Results and Discussion

The results of the numerical calculations, which are shown in Figures 2 and 3, make it evident that the proposed methodology is far less expensive to implement than the existing

Table 4: Crisp dual hesitant fuzzy multiobjective transportation problem by the proposed score function $[K(\widetilde{d})]$.

	D_1	D_2	Supply $(\widetilde{a}_i)$
S_1	0.2 0.2 0.033	0.1 0.067 0.2	5
S_2	0.33 0.23 0.033	0.033 0.133 0.23	3
Demand $(\widetilde{b}_j)$	**6**	**2**	

Note: The bold value represents the demand value of transportation problem.

Table 5: The results of Problem 1 in dual hesitant fuzzy form.

Score function	Methodology	
	Existing methodology [37]	Proposed methodology
Existing score function [28]	(3.55, 2.9, 1.3)	(3.55, 2.9, 1.3)
Proposed score function $[K(\widetilde{d})]$	(1.39, 1.49, 0.66)	(1.39, 1.49, 0.66)

Table 6: Dual hesitant fuzzy multiobjective transportation.

	D_1	D_2	D_3	Supply $(\widetilde{a}_i)$
S_1	$\{(\widetilde{h}_{11})_1,(\widetilde{g}_{11})_1\}$ $\{(\widetilde{h}_{11})_2,(\widetilde{g}_{11})_2\}$ $\{(\widetilde{h}_{11})_3,(\widetilde{g}_{11})_3\}$	$\{(\widetilde{h}_{12})_1,(\widetilde{g}_{12})_1\}$ $\{(\widetilde{h}_{12})_2,(\widetilde{g}_{12})_2\}$ $\{(\widetilde{h}_{12})_3,(\widetilde{g}_{12})_3\}$	$\{(\widetilde{h}_{13})_1,(\widetilde{g}_{13})_1\}$ $\{(\widetilde{h}_{13})_2,(\widetilde{g}_{13})_2\}$ $\{(\widetilde{h}_{13})_3,(\widetilde{g}_{13})_3\}$	**10**
S_2	$\{(\widetilde{h}_{21})_1,(\widetilde{g}_{21})_1\}$ $\{(\widetilde{h}_{21})_2,(\widetilde{g}_{21})_2\}$ $\{(\widetilde{h}_{21})_3,(\widetilde{g}_{21})_3\}$	$\{(\widetilde{h}_{22})_1,(\widetilde{g}_{22})_1\}$ $\{(\widetilde{h}_{22})_2,(\widetilde{g}_{22})_2\}$ $\{(\widetilde{h}_{22})_3,(\widetilde{g}_{22})_3\}$	$\{(\widetilde{h}_{23})_1,(\widetilde{g}_{23})_1\}$ $\{(\widetilde{h}_{23})_2,(\widetilde{g}_{23})_2\}$ $\{(\widetilde{h}_{23})_3,(\widetilde{g}_{23})_3\}$	**10**
S_3	$\{(\widetilde{h}_{31})_1,(\widetilde{g}_{31})_1\}$ $\{(\widetilde{h}_{31})_2,(\widetilde{g}_{31})_2\}$ $\{(\widetilde{h}_{31})_3,(\widetilde{g}_{31})_3\}$	$\{(\widetilde{h}_{32})_1,(\widetilde{g}_{32})_1\}$ $\{(\widetilde{h}_{32})_2,(\widetilde{g}_{32})_2\}$ $\{(\widetilde{h}_{32})_3,(\widetilde{g}_{32})_3\}$	$\{(\widetilde{h}_{33})_1,(\widetilde{g}_{33})_1\}$ $\{(\widetilde{h}_{33})_2,(\widetilde{g}_{33})_2\}$ $\{(\widetilde{h}_{33})_3,(\widetilde{g}_{33})_3\}$	**40**
Demand $(\widetilde{b}_j)$	**20**	**15**	**25**	

Note: The bold values represent the crisp supply and demand quantities that ensure feasibility of the transportation problem.

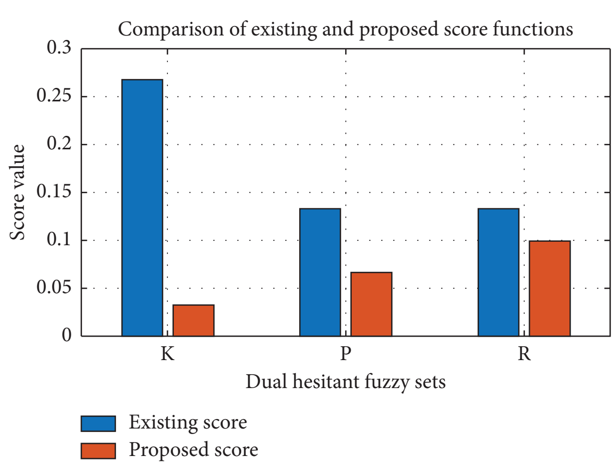

Figure 2: Graphical representation of the comparison of the existing and proposed score functions.

one. First, we applied the existing score function with the already developed methodology and the proposed methodology in DHFMOTP. When the existing methodology was combined with a purpose-built score function, the cost was reduced from (3.55, 2.9, 1.3) to (1.39, 1.49, 0.66) and from (14.5, 14.01, 11.66) to (3.75, 2.83, 1.75), respectively. Using

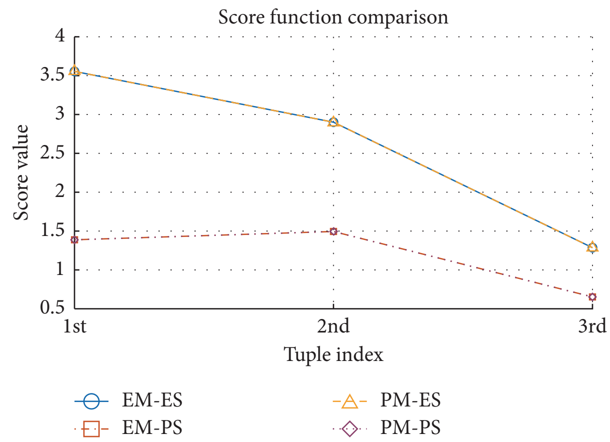

Figure 3: Comparison of costs for Problem 1.

Table 7: Crisp form of Dual hesitant fuzzy multiobjective transportation Problem 2 by existing score function.

	D_1	D_2	D_3	Supply $(\tilde{a}_i)$
S_1	0.267 0.133 0.133	0.20 0.133 0.10	0.267 0.233 0.20	10
S_2	0.1 0.133 0.1	0.10 0.30 0.167	0.133 0.33 0.167	10
S_3	0.30 0.27 0.366	0.20 0.23 0.2	0.30 0.30 0.20	40
Demand $(\tilde{b}_j)$	**20**	**15**	**25**	

Note: The bold values represent the crisp demand of the transportation problem.

Table 8: Crisp dual hesitant fuzzy multiobjective transportation by the proposed score function $[K(\tilde{d})]$.

	D_1	D_2	D_3	Supply $(\tilde{a}_i)$
S_1	0.033 0.067 0.10	0.033 0.0167 0.05	0.067 0.05 0.0167	10
S_2	0.067 0.05 0.0167	0.033 0.0167 0.05	0.05 0.1 0.033	10
S_3	0.083 0.0167 0.033	0.0167 0.033 0.067	0.067 0.083 0.0167	40
Demand $(\tilde{b}_j)$	**20**	**15**	**25**	

Note: The bold values represent the crisp demand quantities of the transportation problem.

Table 9: The results of Problem 2 in dual hesitant fuzzy form.

Score function	**Methodology**	
	Existing methodology [37]	**Proposed methodology**
Existing score function [28]	(14.5, 14.01, 11.66)	(14.17, 13.61, 10.33)
Proposed score function $[K(\tilde{d})]$	(3.75, 2.83, 1.75)	(3.75, 2.83, 1.75)

the proposed methodology with the existing score function and the proposed methodology with the intended scoring function in Problems 1 and 2, respectively, we were able to reduce the costs from (3.55, 2.9, 1.3) to 1.39, 1.49, 0.66) and from (14.17, 13.61, 10.33) to (3.75, 2.83, 1.75). Subsequently, we see that our algorithm and scoring function provide us with the most optimal shipping cost. This consistency shows that the proposed score function maintains reliability across

Table 10: Comparison between the existing and proposed score functions.

Score functions	Dual hesitant fuzzy sets		
	$\widehat{K} = \{(0.5, 0.4, 0.1)$ $(0.4, 0.5, 0.9)\}$	$\widehat{P} = \{(0.4, 0.2, 0.3)$ $(0.1, 0.8, 0.4)\}$	$\widehat{R} = \{(0.1, 0.2, 0.4)$ $(0.6, 0.3, 0.2)\}$
Existing score function	0.267	0.133	0.133
Proposed score function	0.033	0.067	0.10

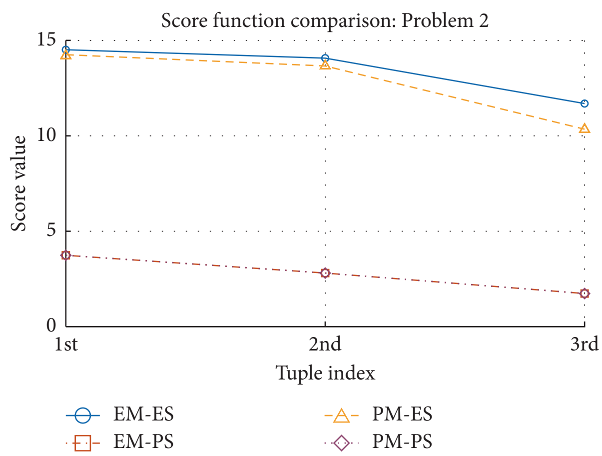

Figure 4: Comparison of costs for Problem 2. EM-ES: existing method, existing score; EM-PS: existing method, proposed score; PM-ES: proposed method, existing score; PM-PS: proposed method, proposed score.

different methods and also shows its robustness in performance evaluation. The comparative analysis between the existing and proposed methodologies, focusing on their respective score functions, is shown in Tables 5 and 9. It appears that the suggested score function could provide a strong basis for future advancements given its consistent performance in both approaches.

Table 10 shows the comparative analysis based on the existing and proposed score functions.

7. Conclusion

In conclusion, this study presents a novel approach to address the uncertainties in MOTP by utilizing DHFS. When it comes to maximizing TCs, the proposed score function and methodology, which were specifically developed for DHFMOTP, perform better than the existing scoring functions and approaches. In summary, the innovative methods for addressing MOTP within a DHF environment represent a significant progression in decision-making procedures. Through the utilization of DHFS, this technique adeptly captures the inherent uncertainty and intricacy present in real-world transportation scenarios. In addition, we get more decreasing score values by the proposed score function as compared to the existing score function as shown in Table 10, and also the cost decreased from (3.55,2.9,1.3) to (1.39,1.49,0.66) and from (14.5,14.01,11.66) to (3.75,2.83,1.75) when the current methodology was coupled with a specially designed score function. For Problems 1 and 2, we were able to lower the expenses from (3.55,2.9,1.3) to 1.39,1.49,0.66) and from (14.17,13.61,10.33) to (3.75,2.83,1.75) by using the suggested technique with the intended scoring function and the suggested methodology with the existing score function, respectively, and it can be easily observed from Figure 4. It enables the simultaneous optimization of multiple objectives, such as cost, time, and EI, while allowing for the flexible representation of decision-makers' preferences. This approach not only improves solution precision but also establishes a more resilient framework for tackling the difficulties encountered in logistics and supply chain management. Subsequent research could delve deeper into its potential applications across diverse industries, laying the groundwork for more effective and adaptable transportation strategies.

This work is limited in assuming the constant conditions across the various test scenarios and potentially oversimplifying the real-world complexities in transportation models. Here, for applying the proposed score function, we need to ensure that the condition $|\sum_{i=1}^{M} i\widetilde{h}_{\widetilde{a}}(p) - \sum_{i=1}^{M}\widetilde{g}_{\widetilde{a}}(p)| \ngtr 1$ or $\sum_{i=1}^{M}\widetilde{h}_{\widetilde{a}}(p) < \sum_{i=1}^{M}\widetilde{g}_{\widetilde{a}}(p)$ must be satisfied.

7.1. Future Scope of This Research

- To enhance decision-making by developing hybrid models that integrate DHF methods with other optimization techniques.
- Further investigation into geographic information systems (GISs) can be combined with the DHF approach to enhance spatial decision-making in transportation planning.
- In future, we can also develop the advanced models that can accurately represent the transportation choices of users by incorporating DHF sets to capture their preferences and behaviors.
- The validation of the DHF approach in practical transportation scenarios can be conducted in real-world case studies.

Nomenclature

DHF	Dual hesitant fuzzy
MOTP	Multiobjective transportation problem
DHPF	Dual hesitant Pythagorean fuzzy
FS	Fuzzy set
FTP	Fuzzy transportation problem
HFS	Hesitant fuzzy set
IFS	Intuitionistic fuzzy set

IVFS Interval valued fuzzy set
MD Membership degree
MOTP Multiobjective transportation problem
NMD Nonmembership degree
PHFS Pythagorean hesitant fuzzy set
TP Transportation problem
TC Transportation cost
TT Transportation time

Data Availability Statement

The corresponding author can provide the data supporting the study's conclusions upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest.

Author Contributions

Ritu: methodology, numerical analysis, software, data curation, and writing.

Tarun Kumar: visualization, investigation, writing–review and editing.

Jahnvi: visualization and literature review.

Kapil Kumar: analysis, calculation verification, and validation.

Kailash Dhanuk: problem identification and development of methodology.

Anirudh Kumar Bhargava: interpreting the results and refining the manuscript.

Sanjay Kumar Tyagi: review, data analysis, editing, validation.

M. K. Sharma: supervision, conceptualization, mathematical formulation, original draft, review, editing, and validation.

Funding

No funding was received for this study.

References

[1] F. L. Hitchcock, "The Distribution of a Product From Several Sources to Numerous Localities," *Journal of Mathematics and Physics* 20, no. 1–4 (1941): 224–230, https://doi.org/10.1002/sapm1941201224.

[2] G. Maity and S. K. Roy, "Solving a Multi-Objective Transportation Problem With Nonlinear Cost and Multi-Choice Demand," *International Journal of Management Science and Engineering Management* 11, no. 1 (2016): 62–70, https://doi.org/10.1080/17509653.2014.988768.

[3] L. A. Zadeh, "Fuzzy Sets," *Information and Control* 8, no. 3 (1965): 338–353, https://doi.org/10.1016/S0019-9958(65)90241-X.

[4] K. T. Atanassov, "Intuitionistic Fuzzy Sets," *Physica-Verlag HD* 20 (1999): https://doi.org/10.1016/S0165-0114(86)80034-3.

[5] V. Torra, "Hesitant Fuzzy Sets," *International Journal of Intelligent Systems* 25, no. 6 (2010): 529–539, https://doi.org/10.1002/int.20418.

[6] B. Zhu, Z. Xu, and M. Xia, "Distance and Similarity Measures for Hesitant Fuzzy Sets," *Information Sciences* 181, no. 11 (2011): 2128–2138, https://doi.org/10.1016/j.ins.2011.01.028.

[7] S. Chanas, W. Kołodziejczyk, and A. Machaj, "A Fuzzy Approach to the Transportation Problem," *Fuzzy Sets and Systems* 13, no. 3 (1984): 211–221, https://doi.org/10.1016/0165-0114(84)90057-5.

[8] P. Singh, "A New Method for Solving Dual Hesitant Fuzzy Assignment Problems With Restrictions Based on Similarity Measure," *Applied Soft Computing* 24 (2014): 559–571, https://doi.org/10.1016/j.asoc.2014.08.008.

[9] A. Kumar, S. S. Appadoo, and P. Kaur, "Mehar Approach for Solving Dual-Hesitant Fuzzy Transportation Problem With Restrictions," *Sādhanā* 45 (2020): 77–79, https://doi.org/10.1007/s12046-020-01325-1.

[10] A. Y. Adhami and F. Ahmad, "Interactivepythagorean-Hesitant Fuzzy Computational Algorithm for Multi-objective Transportation Problem Under Uncertainty," *International Journal of Management Science and Engineering Management* 15, no. 4 (2020): 288–297, https://doi.org/10.1080/17509653.2020.1783381.

[11] A. Saranya and J. M. Vinotha, "Dual Hesitant Multi Objective Fractional Transportation Problem With Non-Linear Discount Cost," *International Journal of Aquatic Science* 12, no. 2 (2021): 3180–3191.

[12] F. Ahmad, A. Y. Adhami, B. John, and A. Reza, "A Novel Approach for the Solution of Multiobjective Optimization Problem Using Hesitant Fuzzy Aggregation Operator," *RAIRO-Operations Research* 56, no. 1 (2022): 275–292, https://doi.org/10.1051/ro/2022006.

[13] F. J. Santos-Arteaga, D. Caprio, M. Tavana, and E. C. Tena, "A Credibility and Strategic Behavior Approach in Hesitant Multiple Criteria Decision-Making With Application to Sustainable Transportation," *IEEE Transactions on Fuzzy Systems* 31, no. 2 (2023): 460–474, https://doi.org/10.1109/TFUZZ.2022.3188875.

[14] S. Ghosh, K. H. Kufer, S. K. Roy, and G. W. Weber, "Carbon Mechanism on Sustainable Multi-Objective Solid Transportation Problem for Waste Management in Pythagorean Hesitant Fuzzy Environment," *Complex & Intelligent Systems* 8, no. 5 (2022): 4115–4143, https://doi.org/10.1007/s40747-022-00686-w.

[15] M. Farnam and M. Darehmiraki, "Supply Chain Management Problem Modelling in Hesitant Fuzzy Environment," *Journal of fuzzy extension and applications* 3, no. 4 (2022): 317–336, https://doi.org/10.22105/jfea.2022.33757.1216.

[16] M. K. Sharma, S. Chaudhary, L. Rathour, and V. N. Mishra, "A Time-Sequential Probabilistic Hesitant Fuzzy Approach to a 3-Dimensional Green Transportation System," in *International Conference on Mathematical Modelling, Applied Analysis and Computation* (Springer Nature Switzerland, 2023), 143–156, https://doi.org/10.1007/978-3-031-56304-1_9.

[17] T. C. Koopmans, "Optimum Utilization of the Transportation System," *Econometrica* 17 (1949): 136–146, https://doi.org/10.2307/1907301.

[18] S. M. Chen and L. W. Lee, "Fuzzy Decision-Making Based on Likelihood-Based Comparison Relations," *IEEE Transactions on Fuzzy Systems* 18, no. 3 (2010): 613–628, https://doi.org/10.1109/TFUZZ.2010.2045385.

[19] S. M. Chen, M. W. Yang, S. W. Yang, T. W. Sheu, and C. J. Liau, "Multicriteria Fuzzy Decision Making Based on Interval-Valued Intuitionistic Fuzzy Sets," *Expert Systems with Applications* 39, no. 15 (2012): 12085–12091, https://doi.org/10.1016/j.eswa.2012.04.021.

[20] D. R. Mahapatra, S. K. Roy, and M. P. Biswal, "Multi-Choice Stochastic Transportation Problem Involving Extreme Value Distribution," *Applied Mathematical Modelling* 37, no. 4 (2013): 2230–2240, https://doi.org/10.1016/j.apm.2012.04.024.

[21] P. Kundu, S. Kar, and M. Maiti, "Fixed Charge Transportation Problem With Type-2 Fuzzy Variables," *Information Sciences* 255 (2014): 170–186, https://doi.org/10.1016/j.ins.2013.08.005.

[22] J. Ye, "Improved Cross Entropy Measures of Single Valued Neutrosophic Sets and Interval Neutrosophic Sets and Their Multicriteria Decision Making Methods," *Cybernetics and Information Technologies* 15, no. 4 (2015): 13–26, https://doi.org/10.1515/cait-2015-0051.

[23] S. M. Chen and C. H. Chang, "Fuzzymultiattribute Decision Making Based on Transformation Techniques of Intuitionistic Fuzzy Values and Intuitionistic Fuzzy Geometric Averaging Operators," *Information Sciences* 352-353 (2016): 133–149, https://doi.org/10.1016/j.ins.2016.02.049.

[24] G. Maity, S. K. Roy, and J. L. Verdegay, "Multi-Objective Transportation Problem With Cost Reliability Under Uncertain Environment," *International Journal of Computational Intelligence Systems* 9, no. 5 (2016): 839–849, https://doi.org/10.1080/18756891.2016.1237184.

[25] D. Rani and T. R. Gulati, "Uncertain Multi-Objective Multi-Product Solid Transportation Problems," *Sādhanā* 41, no. 5 (2016): 531–539, https://doi.org/10.1007/s12046-016-0491-x.

[26] S. K. Roy, A. Ebrahimnejad, J. L. Verdegay, and S. Das, "New Approach for Solving Intuitionistic Fuzzy Multi-Objective Transportation Problem," *Sādhanā* 43 (2018): 3–12, https://doi.org/10.1007/s12046-017-0777-7.

[27] L. Kaur, M. Rakshit, and S. Singh, "A New Approach to Solve Multi-Objective Transportation Problem," *Applications and Applied Mathematics: International Journal* 13, no. 1 (2018): 10.

[28] G. Maity, D. Mardanya, S. K. Roy, and G. W. Weber, "A New Approach for Solving Dual-Hesitant Fuzzy Transportation Problem With Restrictions," *Sādhanā* 44, no. 4 (2019): 75, https://doi.org/10.1007/s12046-018-1045-1.

[29] M. Tang, J. Wang, J. Lu, G. Wei, C. Wei, and Y. Wei, "Dual Hesitant Pythagorean Fuzzy Heronian Mean Operators in Multiple Attribute Decision Making," *Mathematics* 7, no. 4 (2019): 344, https://doi.org/10.3390/math7040344.

[30] A. K. Nishad and Abhishek, "A New Ranking Approach for Solving Fully Fuzzy Transportation Problem in Intuitionistic Fuzzy Environment," *Journal of Control, Automation and Electrical Systems* 31, no. 4 (2020): 900–911, https://doi.org/10.1007/s40313-020-00611-x.

[31] M. K. Sharma, N. D. Kamini, and N. Dhiman, "A Fuzzy Optimization Technique for Multi-Objective Aspirational Level Fractional Transportation Problem," *Symmetry* 13, no. 8 (2021): 1465, https://doi.org/10.3390/sym13081465.

[32] S. Midya, S. K. Roy, and V. F. Yu, "Intuitionistic Fuzzy Multi-Stage Multi-Objective Fixed-Charge Solid Transportation Problem in a Green Supply Chain," *International Journal of Machine Learning and Cybernetics* 12, no. 3 (2021): 699–717, https://doi.org/10.1007/s13042-020-01197-1.

[33] S. Ghosh, S. K. Roy, A. Ebrahimnejad, and J. L. Verdegay, "Multi-Objective Fully Intuitionistic Fuzzy Fixed-Charge Solid Transportation Problem," *Complex & Intelligent Systems* 7, no. 2 (2021): 1009–1023, https://doi.org/10.1007/s40747-020-00251-3.

[34] M. Niksirat, "A New Approach to Solve Fully Fuzzy Multi-Objective Transportation Problem," *Fuzzy Information and Engineering* 14, no. 4 (2022): 456–467, https://doi.org/10.1080/16168658.2022.2152836.

[35] L. Zhou, S. Chaudhary, M. K. Sharma, A. Dhaka, and A. Nandal, "Artificial Neural Network Dual Hesitant Fermatean Fuzzy Implementation in Transportation of COVID-19 Vaccine," *Journal of Organizational and End User Computing* 35, no. 2 (2023): 1–23, https://doi.org/10.4018/JOEUC.321169.

[36] K. Jabeen, Q. Khan, K. Ullah, T. Senapati, and S. Moslem, "An Approach to MADM Based on Aczel-Alsina Power Bonferroni Aggregation Operators for q-Rung Orthopair Fuzzy Sets," *IEEE Access* 11 (2023): 105248–105261, https://doi.org/10.1109/access.2023.3270267.

[37] K. P. O. Niluminda and E. Ekanayake, "The Multi-Objective Transportation Problem Solve With Geometric Mean and Penalty Methods," *Indonesian Journal of Innovation and Applied Sciences (IJIAS)* 3, no. 1 (2023): 74–85, https://doi.org/10.47540/ijias.v3i1.729.

[38] Q. T. Bui, M. P. Ngo, V. Snasel, W. Pedrycz, and B. Vo, "Information Measures Based on Similarity Under Neutrosophic Fuzzy Environment and Multi-Criteria Decision Problems," *Engineering Applications of Artificial Intelligence* 122 (2023): 106026, https://doi.org/10.1016/j.engappai.2023.106026.

[39] M. K. Sharma and S. Chaudhary, "Dual Hesitant Fuzzy Set in Multi-Objective Transportation Problems in Time Sequence Frame Work," *Applied Soft Computing* 161 (2024): 111777, https://doi.org/10.1016/j.asoc.2024.111777.

[40] S. Chaudhary, T. Kumar, H. Yadav, A. K. Malik, and M. K. Sharma, "Time-Sequential Probabilistic Fermatean Hesitant Approach in Multi-Objective Green Solid Transportation Problems for Sustainable Enhancement," *Alexandria Engineering Journal* 87 (2024): 622–637, https://doi.org/10.1016/j.aej.2023.12.045.

[41] G. Shahzadi, S. Siddique, H. Shehzadi, and M. Deveci, "Multi-Attribute Decision-Making Using (p, q)-Rung Orthopair Fuzzy Hamacher Interactive Aggregation Operators," *Granular Computing* 9, no. 3 (2024): 63–20, https://doi.org/10.1007/s41066-024-00473-7.

[42] H. Z. Ibrahim, "Exploring Complex N, m-Rung Orthopair Fuzzy Aggregation Operators for Enhanced Multi-Attribute Decision Making," *Granular Computing* 9, no. 2 (2024): 48, https://doi.org/10.1007/s41066-024-00471-9.

[43] A. Hussain, K. Ullah, H. Garg, and T. Mahmood, "A Novel Multi-Attribute Decision-Making Approach Based on T-Spherical Fuzzy Aczelalsinaheronian Mean Operators," *Granular Computing* 9, no. 1 (2024): 21, https://doi.org/10.1007/s41066-023-00442-6.

[44] H. Garg, "A New Exponential-Logarithm-Based Single-Valued Neutrosophic Set and Their Applications," *Expert Systems with Applications* 238 (2024): 121854, https://doi.org/10.1016/j.eswa.2023.121854.

[45] Ritu, T. Kumar, N. Dhiman, M. Sharma, K. Kumar, and M. K. Sharma, "Novel Pythagorean Fuzzy Score Function to Optimize Fuzzy Transportation Models," *Results in Engineering* 25 (2025): 104048, https://doi.org/10.1016/j.rineng.2025.104048.